A very small intro to CTMC in phylogenetics

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In a Bayesian framework

We are always interested in knowing the posterior distribution

$$P(\theta|D) \propto P(D|\theta)\,P(\theta)$$

Example

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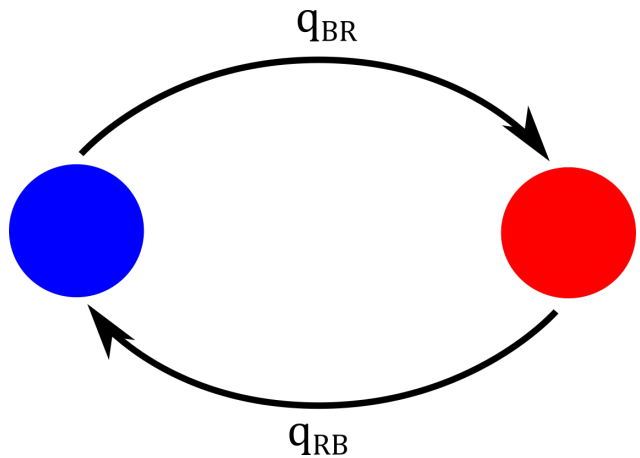

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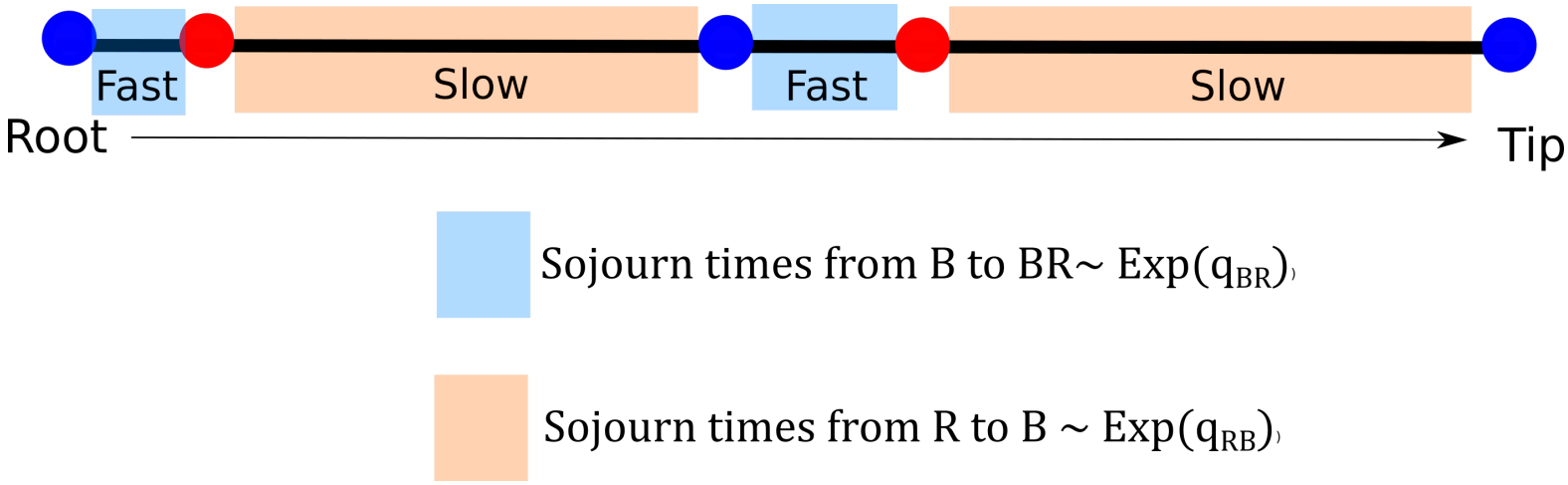
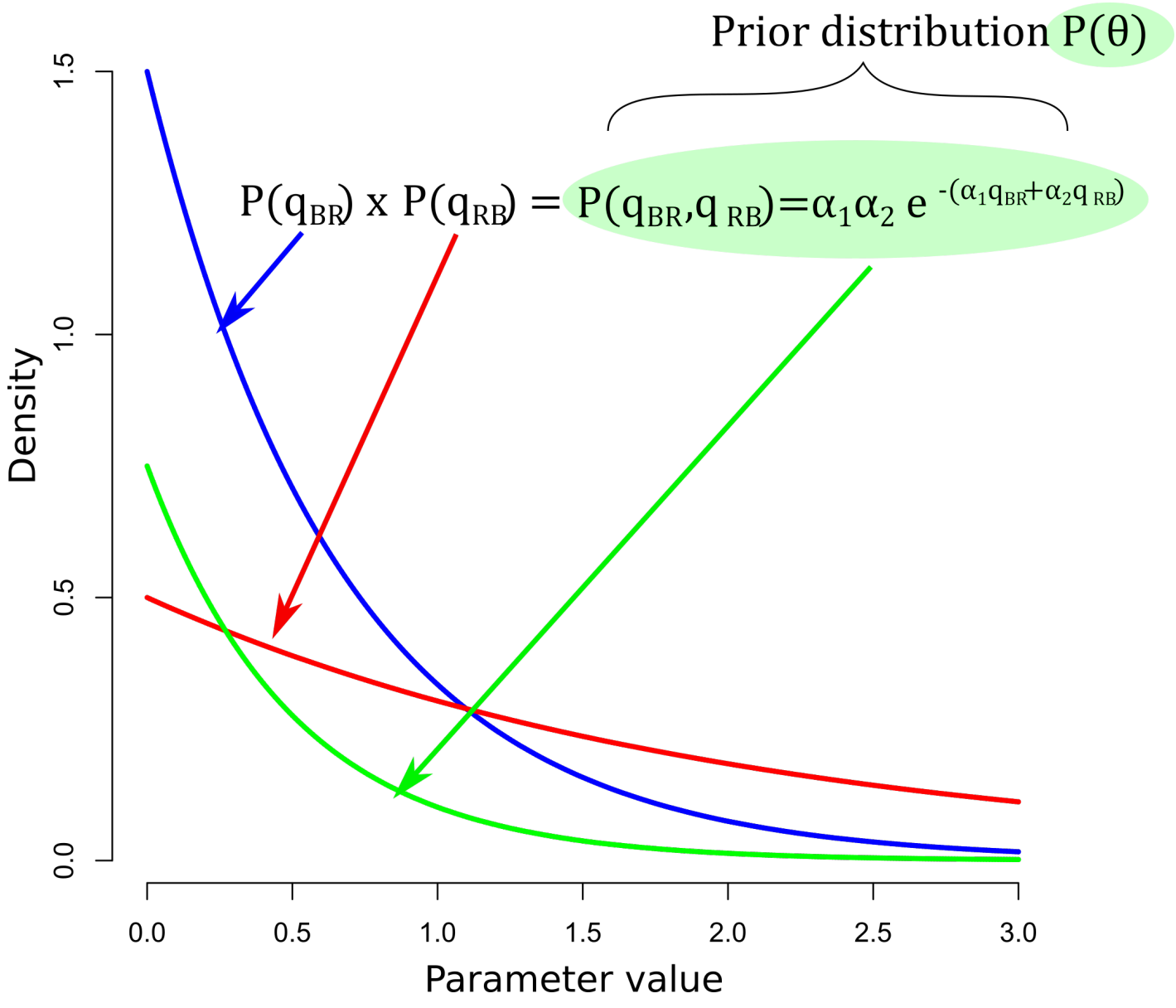
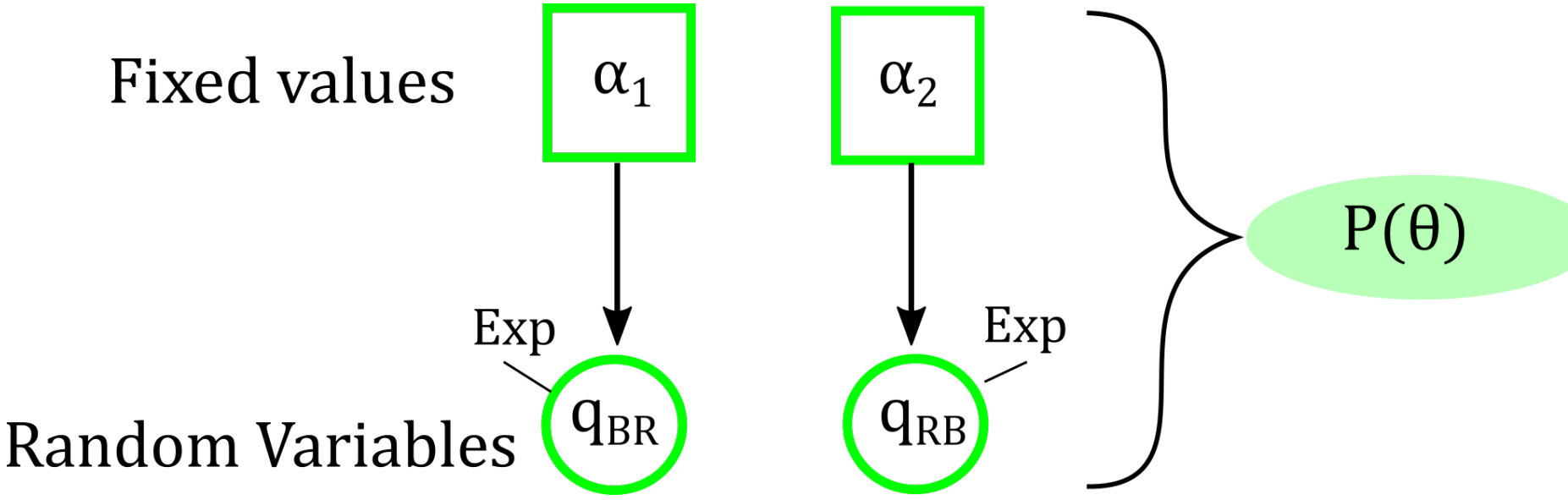


The prior distribution: $P(\theta)$



How are these assumptions represented graphically?



D is our data

We go into our favorite herbarium, field site, or green house and we collect color of multiple species

How do we integrate our model θ and our data D ?

Calculating the likelihood $P(D|\theta)$

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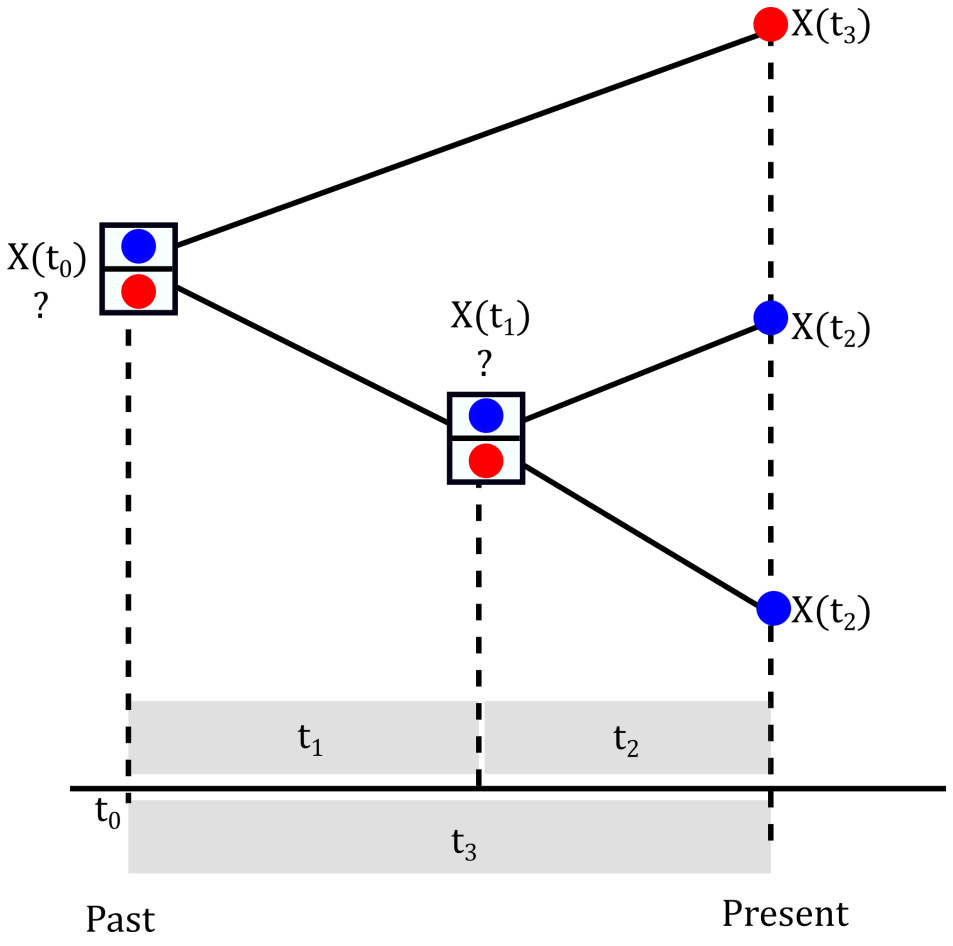
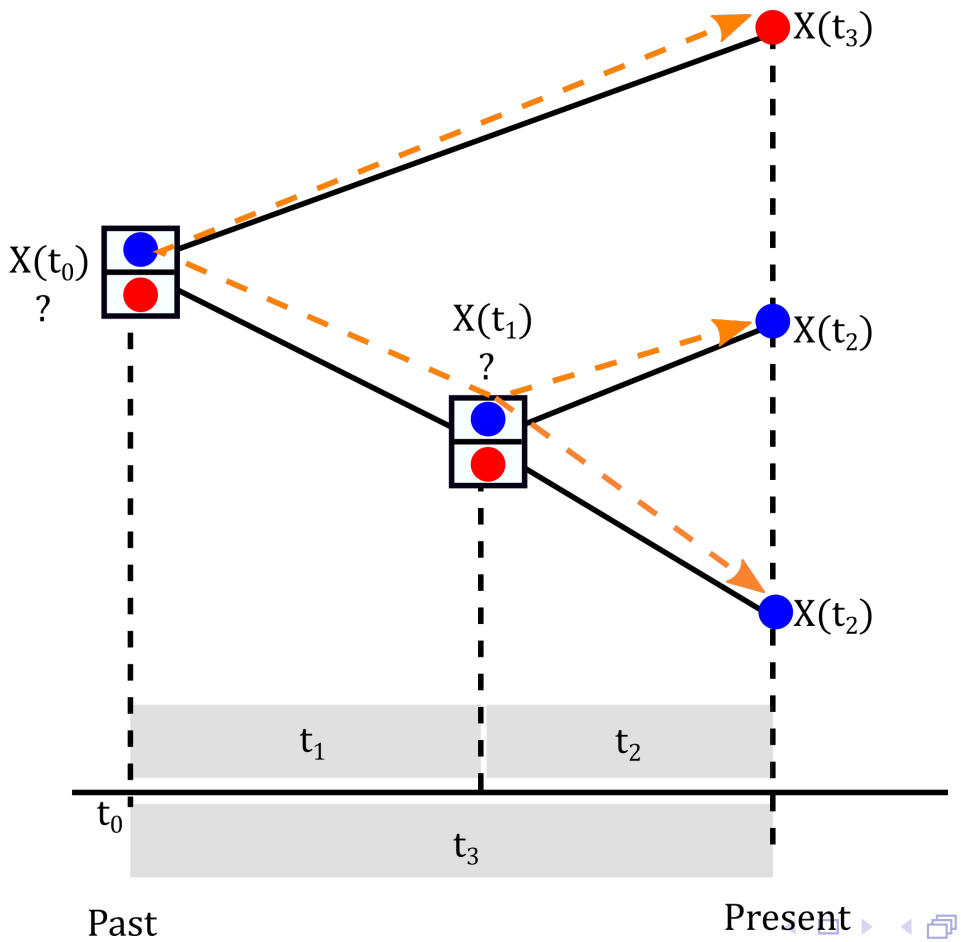
- **Data**: a sample of red and blue flowers on the tips of our phylogeny tree

Likelihood function: The probability of the sample given our hypothesis θ



The probability of a single possible story in phylogenetics

$$P(X(t_2) = B|X(t_1) = B)P(X(t_2) = B|X(t_1) = B)\times$$
$$\times P(X(t_1) = B|X(t_0) = B)P(X(t_3) = R|X(t_0) = B)P(X(t_0) = B)$$



Calculating the likelihood is computationally challenging

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- Reminder: Optimizations to find maximum likelihood estimates and confident intervals require challenging numerical algorithms

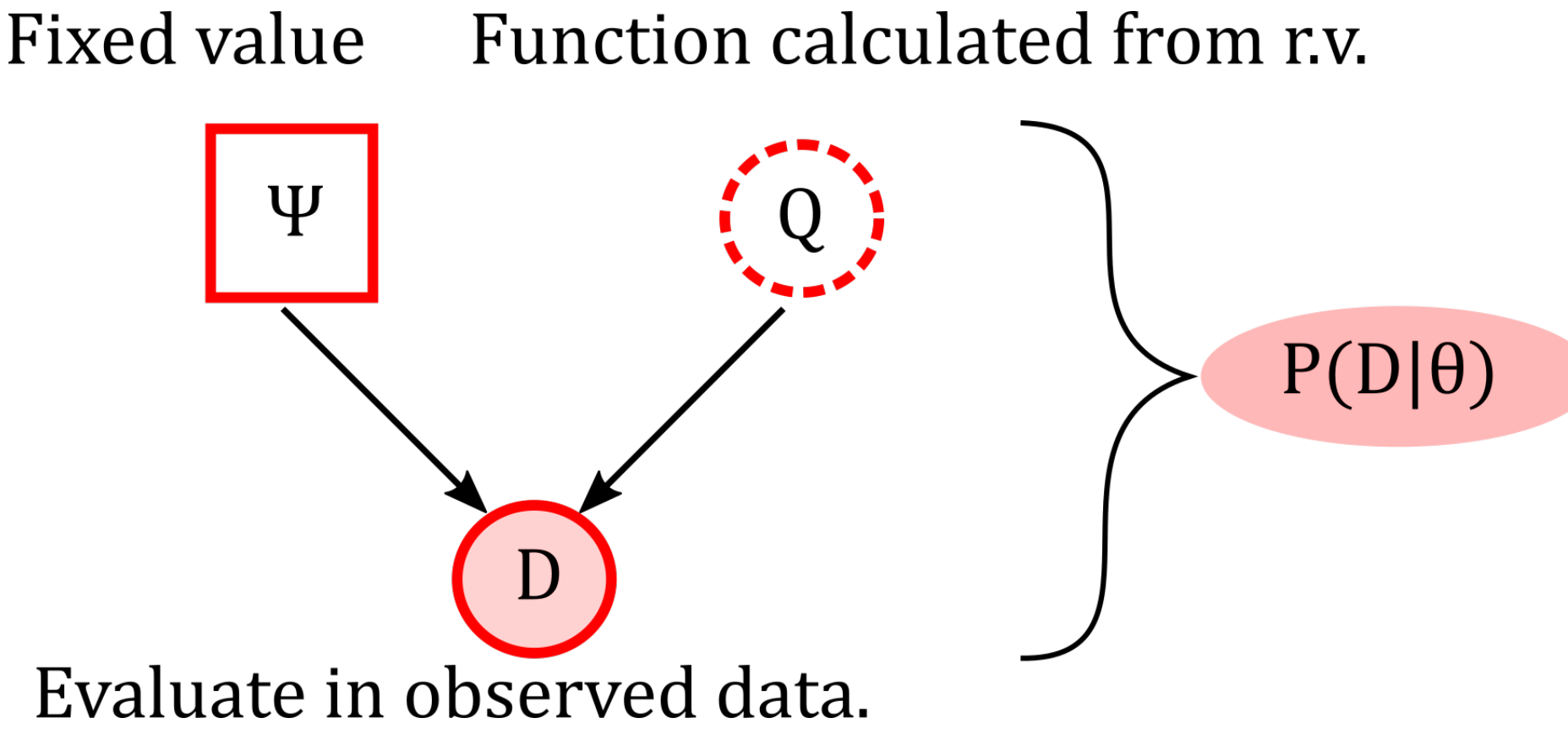
How do the rates connect with the probabilities?

Q-matrix= The infinitesimal probability matrix is the derivative of the probability

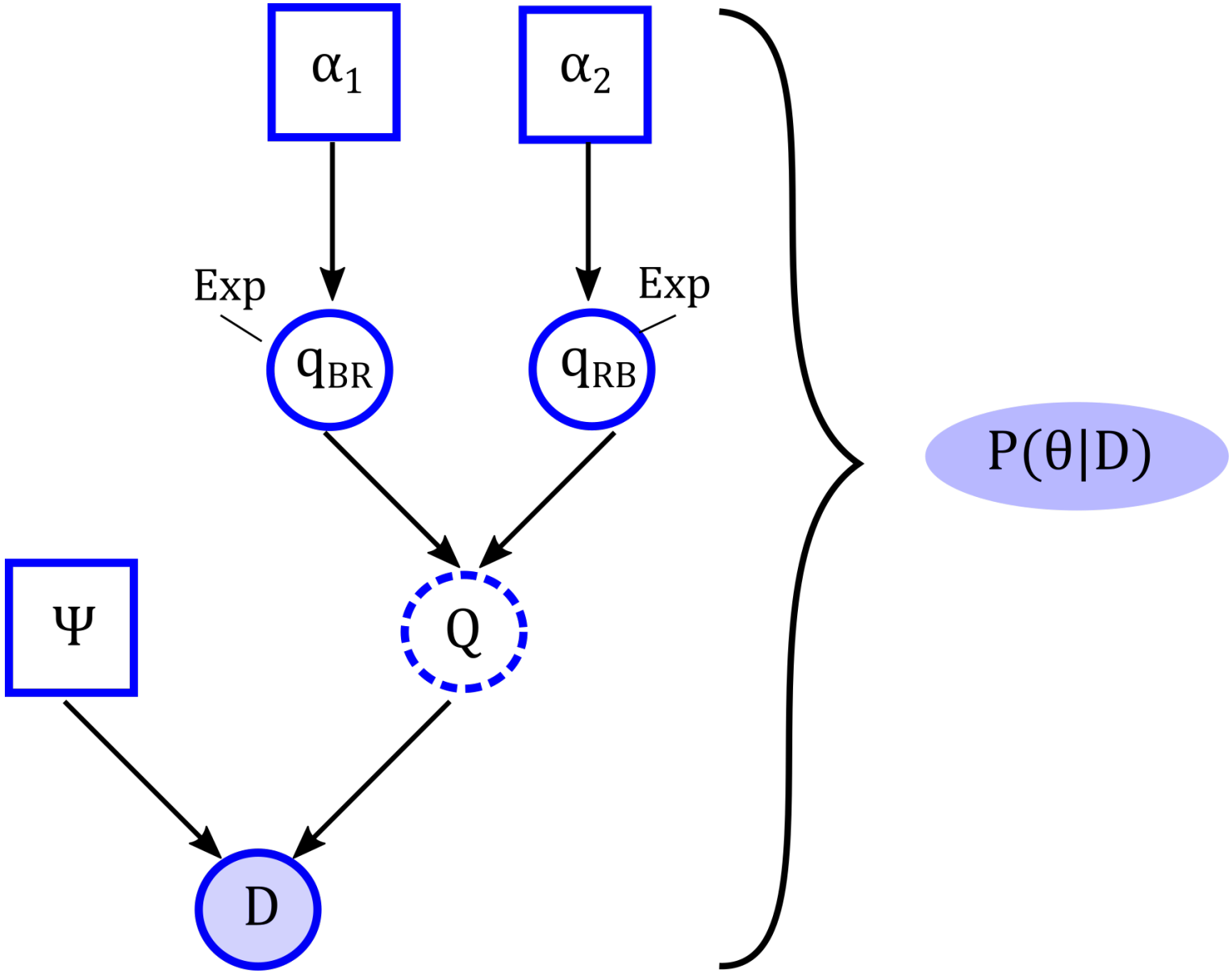
$$\frac{dP(t)}{dt} = Q$$

$$Q = \begin{pmatrix} -q_{BR} & q_{BR} \\ q_{RB} & -q_{RB} \end{pmatrix} \qquad P(t) = e^{Qt}$$

Likelihood in graphical form $P(D|\theta)$



The posterior distribution: the model conditional to the observed data



Graphical model benefits

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- **Explicit notation**: In RevBayes we have notation for fixed variables, random variables, observed data, deterministic function,...
- **Modularity**: Once I have built a model I can connect other as a module (building blocks!)